Formulario Dinámica I & Dinámica II

Cinemática

Coordenadas cartesianas $\vec{r}_i = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ $\quad \vec{v}_i = \dot{x}\,\hat{i} + \dot{y}\,\hat{j} + \dot{z}\,\hat{k}$ $\quad \vec{a}_i = \ddot{x}\,\hat{i} + \ddot{y}\,\hat{j} + \ddot{z}\,\hat{k}$

Coordenadas cilíndricas $\vec{r}_i = r\,\hat{\varepsilon}_r + z\,\hat{k}$ $\quad \vec{v}_i = \dot{r}\,\hat{\varepsilon}_r + r\dot{\theta}\,\hat{\varepsilon}_\theta + \dot{z}\,\hat{k}$ $\quad \vec{a}_i = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\varepsilon}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\varepsilon}_\theta + \ddot{z}\,\hat{k}$

Coordenadas intrínsecas $\vec{r}_i = \vec{r}_i\left(S_{(t)}\right)$ $\quad \vec{v}_i = \dot{S}_{(t)}\,\hat{e}_t$ $\quad \vec{a}_i = \ddot{S}_{(t)}\,\hat{e}_t + \dfrac{\dot{S}_{(t)}^2}{\rho}\,\hat{e}_n$

Sistemas no inerciales

$$\vec{r}_i = \vec{r}_p + \vec{R}_i \quad ; \quad \frac{d\vec{R}_i}{dt} = \vec{\omega}\times\vec{R}_i + \frac{\delta\vec{R}_i}{\delta t} \quad ; \quad \vec{\alpha} = \frac{d\vec{\omega}}{dt} \quad ; \quad \frac{d\hat{I}}{dt} = \vec{\omega}\times\hat{I} \quad \frac{d\hat{J}}{dt} = \vec{\omega}\times\hat{J} \quad \frac{d\hat{K}}{dt} = \vec{\omega}\times\hat{K}$$

$$\vec{v}_i = \vec{v}_p + \vec{\omega}\times\vec{R}_i + \vec{V}_{Ri}$$

$$\vec{a}_i = \vec{a}_p + \vec{\alpha}\times\vec{R}_i + \vec{\omega}\times\left(\vec{\omega}\times\vec{R}_i\right) + 2\,\vec{\omega}\times\vec{V}_{Ri} + \vec{A}_{Ri}$$

$$\vec{a}_i = \vec{a}_p + \vec{\alpha}\times\vec{R}_i - \omega^2\vec{R}_i + 2\,\vec{\omega}\times\vec{V}_{Ri} + \vec{A}_{Ri} \quad \Leftrightarrow \quad (mov.\quad uniplanar)$$

Dinámica de sistemas de partículas

$$M = \sum_{i=1}^{N} m_i \qquad \vec{R}_{CM} = \frac{\sum_{i=1}^{N} m_i\vec{R}_i}{M} \qquad \vec{v}_{CM} = \frac{\sum_{i=1}^{N} m_i\vec{v}_i}{M} \qquad \vec{a}_{CM} = \frac{\sum_{i=1}^{N} m_i\vec{a}_i}{M}$$

$$\vec{p} = \sum_{i=1}^{N}\vec{p}_i = \sum_{i=1}^{N} m_i\vec{v}_i = M\vec{v}_{CM} \qquad \vec{h}_P = \sum_{i=1}^{N}\vec{h}_{Pi} = \sum_{i=1}^{N}\vec{R}_i\times\vec{p}_i = \sum_{i=1}^{N}\vec{R}_i\times m_i\vec{v}_i$$

$$\vec{F}^E = \sum_{i=1}^{N} m_i\vec{a}_i = M\vec{a}_{CM} = \frac{d\vec{p}}{dt} \qquad \vec{M}_P^E = \sum_{i=1}^{N}\vec{R}_i\times m_i\vec{a}_i \qquad \vec{M}_P^E = \frac{d\vec{h}_P}{dt} + \vec{v}_P\times M\vec{v}_{CM}$$

$$\vec{M}_P^E = I_P\vec{\alpha} + \vec{R}_{CM}\times M\vec{a}_P \quad ; \quad I_P = \sum_{i=1}^{N} m_i R_i^2 \quad \Leftrightarrow \quad (mov.\quad uniplanar)$$

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$$W_{1\to2}^{E(nc)} + W_{1\to2}^{I} = \left(T_2 + U_2\right) - \left(T_1 + U_1\right) \quad ; \quad W_{1\to2}^{E(nc)} = \sum_{i=1}^{N}\int_1^2 \vec{F}_i^E \circ d\vec{r}_i \quad W_{1\to2}^{I} = \sum_{i=1}^{N}\int_1^2 \vec{F}_i^I \circ d\vec{r}_i \quad ; \quad U^{grav} = mgh_{ref} \quad U^{elast} = \frac{1}{2}kx^2$$

$$T = \sum_{i=1}^{N} T_i = \sum_{i=1}^{N}\frac{1}{2}m_i v_i^2$$

$$T = \frac{1}{2}Mv_P^2 + \frac{1}{2}I_P\omega^2 + \vec{v}_P\circ\left(\vec{\omega}\times M\vec{R}_{CM}\right) \quad \Leftrightarrow \quad (mov.\quad uniplanar)$$

Dinámica del Cuerpo Rígido

$$\vec{p} = M\vec{v}_{CM} \qquad \vec{h}_P = \vec{H}_P + \vec{R}_{CM}\times M\vec{v}_P \quad \vec{h}_P = \vec{H}_{CM} + \vec{R}_{CM}\times M\vec{v}_{CM} \qquad \vec{H}_P = \mathbf{I}_{PXYZ}\vec{\omega}$$

$$\vec{F}^E = M\vec{a}_{CM} = \frac{d\vec{p}}{dt} \quad ; \quad \vec{M}_P^E = \frac{d\vec{h}_P}{dt} + \vec{v}_P\times M\vec{v}_{CM} \quad ; \quad \vec{M}_P^E = \mathbf{I}_{PXYZ}\vec{\alpha} + \vec{\omega}\times\mathbf{I}_{PXYZ}\vec{\omega} + \vec{R}_{CM}\times M\vec{a}_P \quad \vec{M}_P^E = \mathbf{I}_{CMXYZ}\vec{\alpha} + \vec{\omega}\times\mathbf{I}_{CMXYZ}\vec{\omega} + \vec{R}_{CM}\times M\vec{a}_{CM}$$

$$\vec{M}_P^E = \mathbf{I}_P\vec{\alpha} + \vec{R}_{CM}\times M\vec{a}_P \quad \vec{M}_P^E = \mathbf{I}_{CM}\vec{\alpha} + \vec{R}_{CM}\times M\vec{a}_{CM} \quad \Leftrightarrow \quad (mov.\quad uniplanar)$$

$$\mathbf{I}_{PXYZ} = \mathbf{I}_{CMXYZ} + \mathbf{I}_{CM/PXYZ} \quad ; \quad \mathbf{I}_{CM/PXYZ} = M\begin{bmatrix} Y_{CM}^2 + Z_{CM}^2 & -X_{CM}Y_{CM} & -X_{CM}Z_{CM} \\ & X_{CM}^2 + Z_{CM}^2 & -Y_{CM}Z_{CM} \\ Sym & & X_{CM}^2 + Y_{CM}^2 \end{bmatrix}$$

$$\mathbf{I}_{PX'Y'Z'} = \mathbf{L}^T\mathbf{I}_{PXYZ}\mathbf{L} \quad ; \quad \left\langle \hat{I}\ \hat{J}\ \hat{K}\right\rangle^T = \mathbf{L}\left\langle \hat{I}'\ \hat{J}'\ \hat{K}'\right\rangle^T$$

$$W_{1\to2}^{E(nc)} = \left(T_2 + U_2\right) - \left(T_1 + U_1\right) \quad ; \quad W_{1\to2}^{E(nc)} = \sum_{i=1}^{N}\int_1^2 \vec{F}_i^E \circ d\vec{r}_i \quad ; \quad U^{grav} = mgh_{CMref} \quad U^{elast} = \tfrac{1}{2}kx^2$$

$$T = \frac{1}{2}Mv_P^2 + \frac{1}{2}\vec{\omega}\circ\mathbf{I}_{PXYZ}\vec{\omega} + \vec{v}_P\circ\left(\vec{\omega}\times M\vec{R}_{CM}\right)$$

$$T = \frac{1}{2}Mv_P^2 + \frac{1}{2}I_P\omega^2 + \vec{v}_P\circ\left(\vec{\omega}\times M\vec{R}_{CM}\right) \quad \Leftrightarrow \quad (mov.\quad uniplanar)$$

Ecuaciones de Lagrange

$$\vec{r}_i = \vec{r}_i\left(q_1, q_2, \cdots, q_N\right)$$

$$Q_j^{(nc)} = \frac{d}{dt}\left(\frac{\partial T}{\partial\dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} + \frac{\partial D}{\partial\dot{q}_j} \quad {\scriptstyle j=1,\cdots,N} \quad ; \quad \delta W = \sum_{i=1}^{N} Q_j\,\delta q_j = \sum_{i=1}^{N}\vec{F}_i\circ\delta\vec{r}_i \quad ; \quad D = \frac{1}{2}C\dot{x}^2$$

Teoría de choques

$$\vec{F}^E = \Delta\vec{p} \quad ; \quad \vec{M}_P^E = \Delta\vec{h}_P \quad ; \quad e = \frac{\left|Vel.\,rel.\,alejamiento\ de\ los\ ptos.\,en\ contacto\circ L.I.\right|}{\left|Vel.\,rel.\,aproximacion\ de\ los\ ptos.\,en\ contacto\circ L.I.\right|}$$

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